Effects of the Galilean Moons on Jupiter Flyby Trajectories

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The perturbative effects of the four largest moons of Jupiter on a spacecraft that passes close to Jupiter are investigated using a total of 20 different transfer trajectories. An accurate model for the moons is utilized in the equations of motion which are numerically integrated over the time that the probe spends within Jupiter's sphere of influence. The effects of the moons' perturbations on the postencounter heliocentric trajectories are shown to be large enough to warrant their inclusion in detailed mission simulations, particularly when precision targeting is required (Jupiter swingby to an outer planet, for example).

Introduction

SIGNIFICANT scientific interest has been generated in exploring areas of our solar system that have been uncharted due to the large launch energies required for direct launches from the Earth. These areas include the near-solar regions, the regions above and below the ecliptic plane, and the deep-space regions, as well as the outer planets, Saturn through Pluto. An imaginative way around this problem is to use the gravitational attraction of a planet to modify an interplanetary trajectory. When a probe is in a free-fall trajectory about the Sun in such a manner that it passes within the vicinity of a planet, the trajectory is considerably altered by this encounter. The changes, of course, depend on the encounter geometry of the probe and the planet. Jupiter, being the most massive planet, is ideally suited for this purpose.

Jupiter has twelve natural satellites, four of which are rather large and whose masses are relatively well known. It is the intent of this study to include the perturbative effects of these four moons on Jupiter flyby trajectories. Previous studies[1-6] have neglected the effects of the moons and have treated the problem only in a two-body sense.

The Transfer Trajectory

Since the purpose of the present study is to determine the perturbative effects of the Galilean moons, the transfer trajectory is of only minor interest. For this reason, the complex problem of designing realistic transfer trajectories was simplified to use only time of flight and the positions of the Earth and Jupiter at launch and arrival, respectively, as constraints. This is Lambert's problem, which is readily solved as in Ref. 7 to yield an approximate transfer trajectory.

During the transfer stage, Jupiter and the Earth are considered to be massless; but as the probe nears Jupiter, the concept of a sphere of influence is introduced. The gravitational field of a planet is considered to have a force field whose radius S is given by[8]

$$S = (m/M)^{2/5}R_p \tag{1}$$

where m and M are the masses of the planet and Sun, respectively, and R_p is the planet's distance from the Sun. When the probe reaches this distance from Jupiter, the central body then becomes Jupiter and the probe's position and velocity are now Joviocentric.

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Encounter Geometry with Jupiter

Within Jupiter's sphere of influence the probe's trajectory will be hyperbolic with respect to Jupiter[8] (see Fig. 1). Letting **r** and **v** be the probe's Joviocentric position and velocity, the assumption is made that we may choose the radius of closest approach, r_p, of the probe without significantly altering the direction or magnitude of **v**. This is equivalent to assuming that, while the direction and magnitude of **v** are fixed at the time of arrival at the sphere of influence, we are still free to choose the point at which the probe pierces the surface of this sphere (point A in Fig. 1). The validity of this assumption is indicated by Table 1 which was compiled using patch conic techniques for both retrograde and posigrade flybys of Jupiter and for a radius of closest approach, r_p, varying from 100,000 to 10,000,000 km. These values for r_p span a far larger region than would normally be of interest.

A convenient method for specifying the location of point A on the sphere and the radius of closest approach is to select the magnitude of the miss vector ($|\mathbf{B}| = B$ in Fig. 1) and an angle ψ, measured between **B** and a fixed vector $\mathbf{T}°$ in Jupiter's orbital plane.[1,2,4] The vector $\mathbf{T}°$ is one axis of an orthogonal coordinate system whose origin is Jupiter's center and consists of the unit vectors $\mathbf{S}°$, $\mathbf{T}°$ and $\mathbf{R}°$.[10,11,14,15] This coordinate system is defined as follows:

$$\mathbf{S}° = \mathbf{V}/|\mathbf{V}| \tag{2}$$

$\mathbf{S}°$ is assumed to lie along the incoming asymptote of the hyperbola,

$$\mathbf{T}° = \mathbf{k}° \times \mathbf{S}° \tag{3}$$

where $\mathbf{k}°$ is a unit vector perpendicular to Jupiter's orbital plane,

$$\mathbf{R}° = \mathbf{S}° \times \mathbf{T}° \tag{4}$$

The vector $\mathbf{T}°$ lies in Jupiter's orbital plane and is positive in the direction of Jupiter's motion. We define ψ to be the angle between $\mathbf{T}°$ and **B** measured in the direction $\mathbf{T}°$ to $\mathbf{R}°$. Figure 2 (see Ref. 9) shows the geometry of this coordinate system and its relationship to the impact plane which is perpendicular to the incoming asymptote.

Unperturbed Jupiter Flyby

It can be shown that choosing the magnitude B of the miss vector **B** and ψ (which defines the entrance point on the sphere of influence) completely defines the nominal two-body trajectory about Jupiter under the above assumption that the velocity vector **v** remains unchanged. Let a be the semimajor axis of the hyperbolic orbit. Then we immediately

Table 1 The velocity vector as a function of choosing the entrance point on the sphere of influence

r_p (10^5 km) Posigrade	Right ascension, deg	$\mathbf{V}°$ Declination, deg	$\lvert\mathbf{V}\rvert$ Magnitude, km/sec
1	161.38295	−0.037895218	13.863505
5	161.36287	−0.037826118	13.873625
10	161.34038	−0.037764311	13.882193
50	161.10998	−0.037458739	13.910651
100	160.75975	−0.037179910	13.922712
Retrograde			
1	161.43148	−0.037922481	13.863560
5	161.48549	−0.037896001	13.873828
10	161.53529	−0.037877017	13.882264
50	161.82623	−0.037871506	13.910774
100	162.11483	−0.037958746	13.923062

have

$$a = \mu_p S/(2\mu_p - S|\mathbf{v}|^2) \tag{5}$$

where S is the radius of the sphere of influence. Reference to Fig. 1 yields

$$e = 1/\cos\epsilon \tag{6}$$

and

$$\sin\epsilon = (B/a)\cos\epsilon \tag{7}$$

Therefore

$$\tan\epsilon = B/a \tag{8}$$

The effect of Jupiter on the nominal trajectory is simply to rotate $\mathbf{v}$ through a turning angle γ since energy is conserved.[1,4,8] Gamma (γ) is calculated from

$$\gamma = \pi - 2\epsilon \text{ or, } \gamma = \pi - 2\tan^{-1}(B/a) \tag{9}$$

The radius of closest approach is then

$$r_p = a(1 - e) \tag{10}$$

We have shown that the entrance point on the sphere of influence can be chosen without altering the magnitude and direction of $\mathbf{v}$. The foregoing discussion shows how this point is defined by the parameters B and ψ which completely define the orbit. However, by so choosing B and ψ we obviously change $\mathbf{r}$, the Joviocentric position of the probe. We can use B and ψ to obtain the correct $\mathbf{r}$ and this, along with

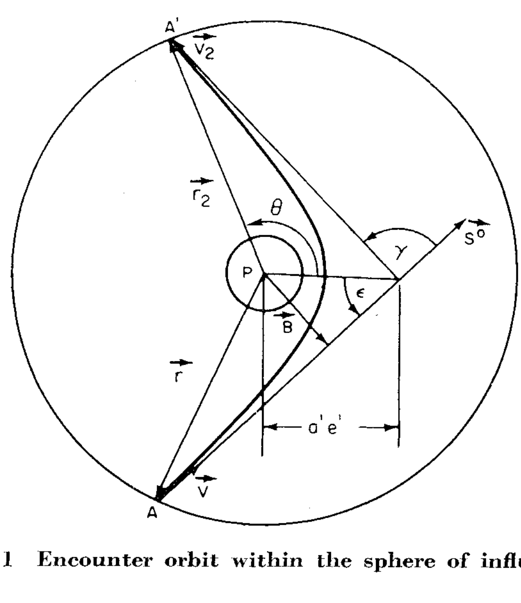

Fig. 1 Encounter orbit within the sphere of influence.

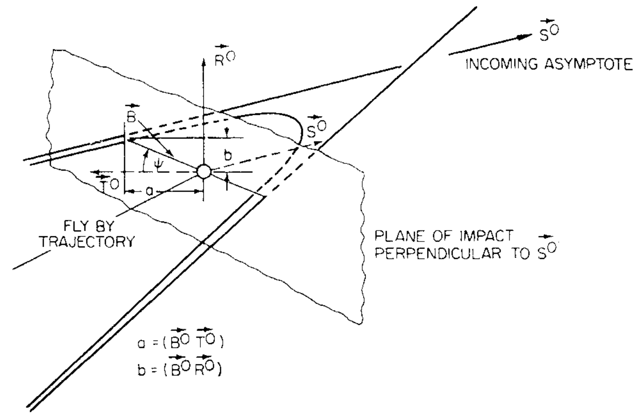

Fig. 2 Planetary approach coordinate system and impact plane.

the already obtained $\mathbf{v}$, becomes the initial condition for numerical integration.

Figure 3 shows the relationship of the vectors and angles involved in the $\mathbf{R}°$, $\mathbf{S}°$, $\mathbf{T}°$ coordinate system. Referring to this figure it is seen that

$$\mathbf{B}° = \mathbf{T}°\cos\psi + \mathbf{R}°\sin\psi \tag{11}$$

From Eqs. (5) and (6)

$$\rho = |a(e^2 - 1)| \tag{12}$$

and the true anomaly at entrance to the sphere is then

$$\theta = \cos^{-1}[(\rho - S)/Se] \tag{13}$$

Referring now to Fig. 1 we find

$$r = S[\cos(\theta + \epsilon - \pi/2)\mathbf{B}° + \cos(\theta + \epsilon)\mathbf{S}°] \tag{14}$$

We have thus seen how choosing B and ψ allows us to vary the encounter conditions with Jupiter and to compute $\mathbf{r}$ and $\mathbf{v}$, the Joviocentric state vector to be used for numerical integration.

Perturbed Jupiter Flyby

At this point we depart significantly from the previously mentioned studies. Using $\mathbf{r}$ and $\mathbf{v}$, one can use a two-body determination to compute the state vector of the probe at exit from the sphere of influence and thereby determine the effects of the encounter with Jupiter on the heliocentric orbit. In the present study, numerical integration is used to propagate $\mathbf{r}$ and $\mathbf{v}$ to the exit from Jupiter's sphere of influence. Cowell's equations of motion for perturbations in rectangular coordinates are used in the integration in the following form:

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{-k^2(M + m)}{r^3}\mathbf{r} + \sum_i k^2 m_i' \left(\frac{\mathbf{r}_i' - \mathbf{r}}{\rho_i^3} - \frac{\mathbf{r}_i'}{\mathbf{r}_i'^3}\right) \tag{15}$$

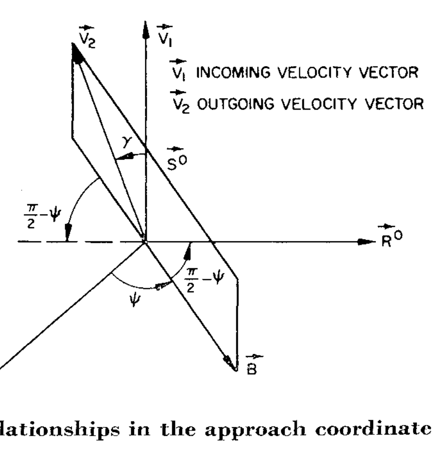

Fig. 3 Relationships in the approach coordinate system.

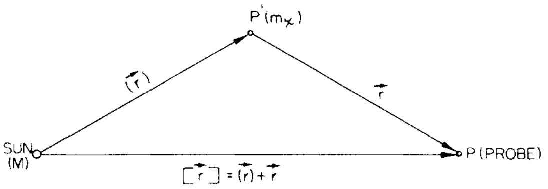

Fig. 4 Vector diagram of Cowell's equations for sun only.

where M and m are the masses of Jupiter and the probe, respectively, $\mathbf{r}$ is the Joviocentric position vector, $\mathbf{r}_i'$ is the position of the ith disturbing body with respect to Jupiter, and $\rho_i = \mathbf{r}_i' - \mathbf{r}$.

In the form of Eq. (15), only the Galilean moons are included as disturbing bodies. However, the Sun must also be included in any realistic study but, because of the Sun's great distance numerical inaccuracies may easily creep into the calculation. For this reason the Sun is handled in the following manner[10]: referring to Fig. 4 and with a slight change in notation, Eq. (15) may be written for the sun only as

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{k^2M♃}{r^3}\mathbf{r} - k^2M\left(\frac{(\mathbf{r})+\mathbf{r}}{|(\mathbf{r})+\mathbf{r}|^3} - \frac{(\mathbf{r})}{|(\mathbf{r})|^3}\right) \quad (16)$$

Now

$$|(\mathbf{r}) + \mathbf{r}|^2 = |(\mathbf{r})|^2 + 2(\mathbf{r})\cdot\mathbf{r} + |\mathbf{r}|^2 \quad (17)$$

Let

$$q = [\tfrac{1}{2}r^2 + (\mathbf{r})\cdot\mathbf{r}]/|(\mathbf{r})|^2$$

Then

$$2|(\mathbf{r})|^2q = r^2 + 2(\mathbf{r})\cdot\mathbf{r}$$

Upon substituting this into Eq. (17), we obtain

$$|(\mathbf{r}) + \mathbf{r}|^2 = |(\mathbf{r})|^2(1 + 2q) \quad (18)$$

Then

$$|(\mathbf{r}) + \mathbf{r}|^{-3} = [1/|(\mathbf{r})|^3](1 + 2q)^{-3/2} \quad (19)$$

Let $h = k^2m♃/r^3$, and

$$(h) = k^2M/|(\mathbf{r})|^3 \quad (20)$$

Substituting Eqs. (19) and (20) into (16) yields

$$d^2\mathbf{r}/dt^2 = -h\mathbf{r} - (h)[(\mathbf{r}) + \mathbf{r}](1 + 2q)^{-3/2} + (h)(\mathbf{r}) \quad (21)$$

or

$$d^2\mathbf{r}/dt^2 = [-h - (h)(1 + 2q)^{-3/2}]\mathbf{r} + (h)(\mathbf{r})[-(1 + 2q)^{-3/2} + 1] \quad (22)$$

Expand the last term into a series

$$1 - (1 + 2q)^{-3/2} = q[3 - (15/2)q + (35/2)q^2 + \ldots] = fq \quad (23)$$

Then Eq. (22) becomes

$$d^2\mathbf{r}/dt^2 = \{(h)fq - (h) - h\}\mathbf{r} + (h)fq(\mathbf{r}) \quad (24)$$

where the term due solely to Jupiter is $h\mathbf{r}$ and the rest is due to the presence of the Sun. Combining now Eqs. (15) and (24) we obtain

$$d^2\mathbf{r}/dt^2 = -h\mathbf{r} + \boldsymbol{\phi} + \mathbf{F} \quad (25)$$

where $-h\mathbf{r}$ is the term for the central body,

$$\boldsymbol{\phi} = \{(h)fq - (h)\}\mathbf{r} + (h)fq(\mathbf{r}) \quad (26)$$

is the term for the Sun and

$$\mathbf{F} = \sum_i k^2m_i\left(\frac{\mathbf{r}_i' - \mathbf{r}}{\rho_i^3} - \frac{\mathbf{r}_i'}{r_i'^3}\right) \quad (27)$$

as defined by Eq. (15) is the acceleration due to the Galilean moons.

Equation (25) is integrated over the time spent by the probe in Jupiter's sphere of influence. The integration technique is a fourth-order predictor-corrector with automatic adjustment of the integration time step.[11]

The integration is initiated at the time of the probe's entrance into the sphere and is terminated at exit from the sphere. This termination is accomplished by checking the probe's distance from Jupiter until this distance becomes equal to the radius of the sphere.

Galilean Satellites of Jupiter

The four great or Galilean satellites of Jupiter were discovered by Galileo Galilei in 1610 and were named by the German astronomer Marius as follows: 1) Io, 2) Europa, 3) Ganymede, and 4) Callisto. Of the twelve known moons of Jupiter these four are by far the largest and they are the only ones whose masses are relatively well known. Of the remaining eight satellites the largest has a diameter of approximately 160 km and the smallest about 15 km.[12] Table 2 is a compilation of the physical data of the Galilean moons and is taken in part from Ref. 13.

When we consider that the planet Mercury has a diameter of 0.38 Earth radii, compared to 0.394 for Ganymede, we see that these moons are by no means small. Figure 5 is a comparative representation of the approach trajectory and the Galilean moons.

These four satellites have been found to be subject to very strong mutual perturbations which cause librations in their orbits about Jupiter. Laplace first discovered a resonance relationship between satellites 1, 2, and 3 which shows that their mean motions are approximately in the proportion of 4:2:1. In terms of mean longitudes $L_1 - 3L_2 + 2L_3 = 180°$.[12] Therefore, the three satellites cannot be in conjunction or opposition at the same time.[13]

Little can be found in the literature on an accurate theory of the motions of the four moons but accurate positions are obviously necessary if a realistic investigation into their effects on the flyby probe is to be made. The theory of the moons is quite complex and useful investigations have only recently begun.

Sampson in 1910[14,15] and DeSitter in 1931[16] have developed detailed theories to describe the motions of the Galilean moons. Sampson's work is perhaps the best known and includes an extensive set of tables which are used to determine the positions of the moons and to predict such physical phenomena as occultations and eclipses. This work which has become known as Sampson's tables represents a monumental and complex task. Although extremely difficult to use, these tables give the times of phenomena to 0.000001 days and positions to 0.000001.[17] Andoyer[18] somewhat simplified the procedure by using only the main terms of Sampson's tables and even though this is less accurate, Andoyer's pro-

Table 2 Astronomical and physical data

	Io	Europa	Ganymede	Callisto
Mass (Earth = 1)	0.0121	0.0079	0.0261	0.0160
Mean diameter (Earth = 1)	0.255	0.226	0.394	0.350
Mean density (H_2O = 1)	4.03	3.78	2.35	2.06
Mean distance from planet, km	421,400	670,500	1,069,500	1,881,200
Sidereal period, days	1.769	3.551	7.155	16.689
Mean orbital velocity, km/sec	17.37	13.77	10.90	8.22
Radius of sphere of influence, km	7,200	9,600	24,800	36,100

cedure has been adopted by the Nautical Almanac Office in their publication of the phenomena of the moons.

More recently Ferraz Mello[19] has studied the planar motion of the Galilean satellites in rectangular coordinates. He shows that the higher-order harmonics of Jupiter's gravitational potential and the relativity corrections are negligible. Marsden[20] has studied the motion of the four moons using Von Zeipel's method to eliminate the short-period terms and to reduce the number of degrees of freedom.

In this study the author used a computerized version of Sampson's tables.† The model is one of a slowly moving plane common to all four moons with inplane perturbations.[21] The model was used to generate an ephemeris tape for the positions of the moons which is used by the trajectory program to avoid the tedious job involved in computing the positions directly from Sampson's tables. Such an ephemerides does not seem to be available elsewhere.

A comparison was made between examples worked out by Sampson in Ref. 14 and the results obtained through use of the ephemeris tape. Table 3 shows the results of this comparison for satellites 1 and 3 on two different epochs. The longitude is measured from the mean equinox of 1900 along the mean ecliptic to the ascending node of Jupiter's orbital plane and thereafter along this plane. The magnitude of the radius vector to the satellite is the projection of the vector into Jupiter's equatorial plane; and the column labeled "tan Latitude" is the tangent of the elevation of the radius vector to Jupiter's orbital plane.

Encounter Geometries and Missions

Twenty different trajectories were studied to determine the effects of the Galilean moons. The launch date for injection into the transfer orbit from Earth to Jupiter was March 8, 1972 which is a favorable date for such a mission. Flight times to Jupiter were varied from 450 to 600 days in steps of 50 days and several combinations of the encounter parameters B and ψ discussed earlier were used. The first twelve cases included flight times of 450, 500, and 550 days. The magnitude of the miss vector B was held constant at 1.25×10^6 km and ψ was varied from 0° to 360° to obtain posigrade, retrograde, and polar encounters with Jupiter. The remaining 8 cases were obtained using a constant value of 180° for ψ (posigrade only) and values of 1.0×10^6 and 1.5×10^6 km for B and included a relatively low energy flight time of 600 days. Those cases where $\psi = 180°$ represent a particularly interesting class of encounter geometries which can gain sufficient energy from Jupiter to become hyperbolic and escape the solar system. Such trajectories also allow one to plan "grand tour" missions to the outer planets.

Those encounter trajectories where $\psi = 0°$ (retrograde) are useful for solar probes since they can result in almost straight-line trajectories and in some cases can "impact"

Table 3 Comparison of Sagnier's method and Sampson's tables (Jovicentered)

	Longitude	Radii, a.u.	tan Latitude
Satellite 1 June 2.156	1909		
Ephemeris	336°07021	0.0000281304	0.018832
Sampson's tables	336°06863	0.0000281296	0.0191815
(difference)	0°00158	8×10^{-10}	0.0003
Satellite 3 April 3.841	1910		
Ephemeris	9°92885	0.00714796	0.045659
Sampson's tables	9°92550	0.00714794	0.045686
(difference)	0°00335	2×10^{-8}	0.000027

† This adaptation of Sampson's tables was received from J. L. Sagnier of Paris, France to whom the author is greatly indebted.

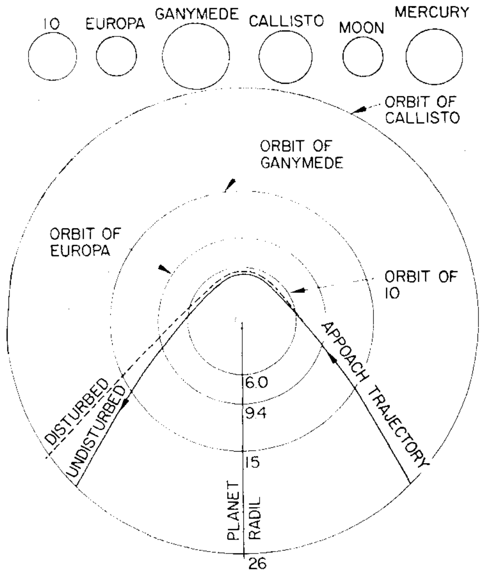

Fig. 5 Typical approach and moon geometry.

the Sun. This cannot be done with a direct flight from the Earth because of the high-energy requirements.[1,4] However, for these trajectories it is desirable that at exit from Jupiter's sphere of influence the heliocentric true anomaly be greater than 180°. Otherwise the total time required to reach the sun is increased prohibitively.

When $\psi = 90°$ the resulting heliocentric orbits are very useful for studies of the solar and galactic media out of the ecliptic plane. A few of the orbits included in this study actually obtain distances in excess of 20 a.u.'s out of the ecliptic, although the total flight times are on the order of dozens of years.

Many other imaginative missions may be undertaken using Jupiter to alter the energy and direction of the heliocentric orbit; however, the present study was limited to only three types of missions and constraints: attaining large distances from the Sun for a deep-space mission in a minimum time; maximum distances out of the ecliptic plane; and close approaches to the Sun when feasible. Figure 6 shows the three phases of transfer, encounter, and post encounter for each of the aforementioned missions.

Results

The effects of the Galilean moons of Jupiter on the heliocentric orbit are presented in Table 4. These results are presented as differences resulting from using only the Sun as a disturbing body and then using the Sun and moons as disturbing bodies. The first three columns identify the case with the parameters B in kilometers, ψ in degrees, and transfer time in days. The next six columns are the differences in the heliocentric Keplerian elements as follows: The semimajor axis (a), eccentricity (e), inclination (i), true anomaly (θ), right ascension of the ascending node (Ω), and argument

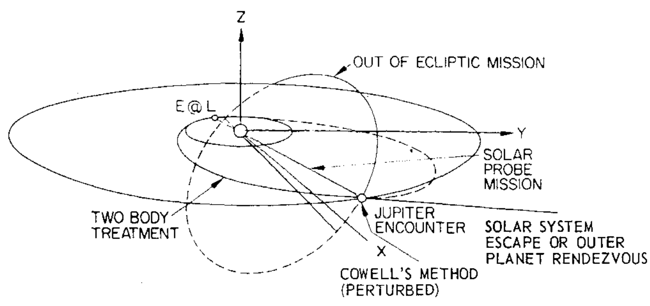

Fig. 6 Mission phases.

Table 4 Effects of the Galilean moons on the heliocentric orbital elements

B, km	$\psi°$	Flight time, days	Δa, km	Δe	$\Delta i°$	$\Delta\theta°$	$\Delta\Omega°$	$\Delta\omega°$	ΔR km[a]	ΔV km/sec	Io, km	Europa, km	Ganymede, km	Callisto, km
1.25×10^6	0	450	310,000	0.0001	0.01	0	0.560	0.52	100,000	0.027	534,000	140,000	156,000	547,000
1.25×10^6	90	450	713,000	0.0033	0.086	0.135	0.081	0.20	50,000	0.024	622,000	815,000	1,270,000	1,140,000
1.25×10^6	180	450	2,510,000	0.0056	0.001	0.148	0.004	0.16	40,000	0.020	453,000	1,130,000	1,710,000	1,510,000
1.25×10^6	270	450	104,700,000	0.0036	0.089	0.141	0.040	0.12	40,000	0.024	582,000	754,000	1,290,000	1,280,000
1.25×10^6	0	500	540,000	0	0.175	0	0.730	0.22	80,000	0.189	464,000	71,100	232,000	88,800
1.25×10^6	90	500	15,400,000	0.0023	0.045	0.005	0.050	0.05	50,000	0.013	489,000	447,000	1,360,000	1,350,000
1.25×10^6	180	500	2,090,000	0.0034	0.003	0.090	0.008	0.11	40,000	0.013	356,000	673,000	1,820,000	1,870,000
1.25×10^6	270	500	23,700,000	0.0026	0.048	0.029	0.020	0.03	40,000	0.014	471,000	380,000	1,340,000	1,440,000
1.25×10^6	0	550	1,730,000	0.0013	0.046	0.021	0.155	0.33	370,000	0.093	338,000	139,000	543,000	435,000
1.25×10^6	90	550	73,500,000	0.0097	0.121	0.279	0.189	0.10	240,000	0.046	801,000	332,000	1,200,000	1,630,000
1.25×10^6	180	550	6,530,000	0.0062	0.053	0.012	0.180	0.19	0	0.027	806,000	6,140	1,490,000	2,230,000
1.25×10^6	270	550	119,800,000	0.0101	0.143	0.290	0.070	0.32	230,000	0.047	823,000	358,000	1,160,000	1,680,000
1.0×10^6	180	450	2,030,000	0.0081	0.001	0.182	0.0049	0.20	90,000	0.026	199,000	787,000	1,530,000	1,310,000
1.5×10^6	180	450	2,990,000	0.0040	0.001	0.127	0.0032	0.13	30,000	0.016	716,000	1,430,000	1,890,000	1,700,000
1.0×10^6	180	500	1,470,000	0.0041	0.003	0.101	0.0099	0.11	70,000	0.014	345,000	255,000	1,610,000	1,690,000
1.5×10^6	180	500	3,000,000	0.0026	0.002	0.079	0.0068	0.09	20,000	0.011	561,000	1,020,000	2,030,000	2,060,000
1.0×10^6	180	550	5,220,000	0.0096	0	0.312	0.0022	0.68	300,000	0.036	794,000	98,800	1,070,000	2,060,000
1.5×10^6	180	550	13,600,000	0.0087	0.001	0.286	0.0036	0.30	140,000	0.036	672,000	274,000	1,810,000	2,390,000
1.0×10^6	180	600	480,000	0.0009	0.005	0.191	0.0148	0.19	230,000	0.004	619,000	764,000	140,000	2,170,000
1.5×10^6	180	600	10,700,000	0.0047	0	0.179	0	0.19	130,000	0.020	1,190,000	803,000	933,000	2,630,000

[a] This column is the differences in helicoentric radial distances and not rss values.

of perihelion (ω). The next two columns are the differences in magnitude of the radius and velocity vectors, respectively. The final four columns show the closest approach to the Galilean moons. The closest approaches to all four moons always occur with a span of 1.5 days and always near perijove of the encounter trajectory where the probe is moving very fast.

A quick glance at the Δa column shows some rather large changes occurring in the semimajor axis which is calculated from the following equation

$$a = r/[2 - (rv^2/\mu)] \tag{28}$$

These large perturbations by virtue of the above equation are attributed to changes in the position and velocity magnitudes. A convenient equation[8] for the rate of change in the semimajor axis can be obtained and used to check the magnitude of the perturbations. Letting $\mathbf{r}(t)$ and $\mathbf{r}_{osc}(t)$ be the true and osculating position vectors, respectively, we define $\mathbf{a}_d = (d^2\mathbf{r}/dt^2) - (d^2\mathbf{r}_{osc}/dt^2)$ then we can derive the following equation:

$$da/dt = (2a^2/\mu)\mathbf{V}\cdot\mathbf{a}_d. \tag{29}$$

Using Eq. (29) and the ephemeris of the moons, numerous hand checks were carried out to verify the values of Δa in Table 4. The checks were done by adding one moon at a time and evaluating da/dt at times near the closest approaches to the moons. Although individually the moons contribution to da/dt was sometimes small, when they were taken together the perturbations became quite significant, indicating a complex dynamical interaction between the bodies. This interaction is difficult to predict analytically and therefore the perturbations cannot be proven conclusively. Further study is under way to verify the variations observed in Table 4. A final word concerning the relative magnitude of these variations should be made. Although Δa appears large in most cases, it must be pointed out that the semimajor axis is itself large. For example case 12 shows a change of 0.12×10^9 km whereas a equals 3×10^9 km, representing a perturbation of 4%. This is the largest percentage of all 20 cases.

The perturbations of the remaining elements as shown in Table 4 exhibit no unusual behavior; however, some comments on the changes occurring in the magnitude of the position vector are needed. Considering the fact that the probe at this point is about 5 a.u. distance from the sun, these changes are small indeed. But if the purpose of the mission is to attain a particular point in space as would be needed for a

Table 5 Effects of the Galilean moons on deep space mission parameters

B, km	ψ, deg.	Flight time, days	Total position deviation at exit from sphere, km[a]	Total velocity deviation at exit from sphere, km/sec[a]	Time in sphere, days	Time to 10 a.u., days	Dist. out of ecliptic, a.u.	Closest approach to sun, km	Io, km	Europa, km	Ganymede, km	Callisto, km
1.25×10^6	0	450	107,000	0.0317	0.001	...	...	...	534,000	140,000	156,000	547,000
1.25×10^6	90	450	110,000	0.0358	0.002	2.84	0.590 0.008	...	622,000	815,000	1,270,000	1,140,000
1.25×10^6	180	450	122,000	0.0375	0.006	0.18	...	...	453,000	1,130,000	1,710,000	1,510,000
1.25×10^6	270	450	110,000	0.0356	0.002	2.75	0.827 0.010	...	582,000	754,000	1,290,000	1,280,000
1.25×10^6	0	500	76,400	0.0194	0.004	...	...	Impact	464,000	71,100	232,000	88,800
1.25×10^6	90	500	82,200	0.0248	0.006	3.40	0.083 0.006	...	489,000	447,000	1,360,000	1,350,000
1.25×10^6	180	500	84,100	0.0226	0.008	0.12	...	...	356,000	673,000	1,820,000	1,870,000
1.25×10^6	270	500	82,100	0.0217	0.006	3.28	0.110 0.006	...	471,000	380,000	1,340,000	1,440,000
1.25×10^6	0	550	416,800	0.0938	0.020	...	...	656,000	338,000	139,000	543,000	435,000
1.25×10^6	90	550	352,800	0.0814	0.006	16.5	0.201 0.010	...	801,000	332,000	1,200,000	1,630,000
1.25×10^6	180	550	146,800	0.0448	0.133	1.49	...	...	806,000	6,140	1,490,000	2,230,000
1.25×10^6	270	550	356,200	0.0821	0.006	14.9	0.002 0.029	...	823,000	358,000	1,160,000	1,680,000
1.0×10^6	180	450	176,800	0.0546	0.004	0.06	...	...	199,000	787,000	1,530,000	1,310,000
1.5×10^6	180	450	92,000	0.0276	0.005	0.21	...	...	716,000	1,430,000	1,890,000	1,700,000
1.0×10^6	180	500	110,000	0.0301	0.005	0	...	...	345,000	255,000	1,610,000	1,690,000
1.5×10^6	180	500	64,400	0.0173	0.009	0.19	...	...	561,000	1,020,000	2,030,000	2,060,000
1.0×10^6	180	550	398,000	0.0895	0.022	1.28	...	...	794,000	98,800	1,070,000	2,060,000
1.5×10^6	180	550	288,700	0.0643	0.012	0.53	...	...	672,000	274,000	1,810,000	2,390,000
1.0×10^6	180	600	268,000	0.0526	0.050	1.11	...	...	619,000	764,000	140,000	2,170,000
1.5×10^6	180	600	210,300	0.0400	0.010	0.21	...	...	1,190,000	803,000	933,000	2,630,000

[a] These deviatons are Jovicentric and are the square root of the sum of the squares of the component deviations.

grand tour mission, then any small deviations become extremely important. For this reason it is more meaningful to look at the total position and velocity deviations of the probe when it exits from Jupiter's sphere of influence. Columns 4 and 5 of Table 5 show the deviations in position and velocity as the square root of the sum of the squares of the component variations. These deviations are Jovicentric and are calculated at exit from the sphere. These differences show that additional requirements may be imposed on the post encounter midcourse guidance dependent on the type of mission desired.

Table 5 also shows the effects of the Galilean moons on three types of missions where guidance may not be of importance. Columns 1, 2, and 3 are again the identifying parameters and columns 4 and 5 have been discussed above. Column 6 is the difference in the time spent in Jupiter's sphere of influence. Columns 7, 8 and 9 are the respective differences in time to reach 10 a.u. from the sun; distances attained out of the ecliptic; and closest approach to the sun where these missions are applicable. Again the last 4 columns are a repetitition of the close approaches to the moons.

Little can be said about the time spent in the sphere and one would not anticipate that the Galilean moons would have an appreciable effect on this parameter. The same can be said about the time to 10 a.u. although as pointed out earlier this point will not be the same point in space that would be reached if one neglected the effects of the moons. Cases 2 and 4 in Table 5 show significant differences occurring in the distances out of the ecliptic; however, the actual distances reached are at least 25 a.u. in both cases, which make the perturbations comparatively small. As pointed out earlier, these distances are impractical because of the time involved to reach them, but such trajectories can be used to study the medium above and below the ecliptic for smaller distances.

Only two cases studied were useful for solar probe studies in that they were the only ones heading back toward the Sun at exit from Jupiter's sphere of influence. The effect of the Galilean moons on this mission is also illustrated in Table 5. The first of these is case 5 and this trajectory impacts the Sun whether the moons are considered or not. The second, case 9, comes within 27.6×10^6 km and the effect of the moons is only about 0.6×10^6 km. Again as with the other missions the moons may be neglected without significantly altering the results.

Conclusions

It would appear from this study that care must be taken in selecting the model to be used for a particular mission utilizing the gravitational benefits of Jupiter. Certainly in all first approximation studies, two-body determinations of a type similar to that outlined in this article or in the references can be used without introducing large errors. However, when final trajectories are to be selected along with midcourse requirements, then it becomes necessary to consider the Galilean moons as disturbing bodies if the mission is actually concerned with achieving a particular point in space. Trajectories for multiple-planet missions or for rendezvous with the comets may be greatly affected by the presence of these moons. On the other hand, those missions whose sole objective is to achieve a desired distance or to achieve a larger inclination to the ecliptic or any mission that involves no specific target point, will not be significantly affected.

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